K. Kunen Set theory Elsevier, 1980

§7. The axioms

For reference, we list here the axioms of ZFC and of some related theories; these are explained in much greater detail in Chapters I and III. After each axiom we list the section in Chapters I or III where it first occurs.

AXIOM 0. *Set Existence* (I §5).

$$\exists x\,(x = x).$$

AXIOM 1. *Extensionality* (I §5).

$$\forall x\,\forall y\,(\forall z\,(z \in x \leftrightarrow z \in y) \rightarrow x = y).$$

AXIOM 2. *Foundation* (III §4).

$$\forall x\,[\exists y\,(y \in x) \rightarrow \exists y\,(y \in x \wedge \neg \exists z\,(z \in x \wedge z \in y))].$$

AXIOM 3. *Comprehension Scheme* (I §5). For each formula ϕ with free variables among $x, z, w_1, \ldots, w_n$,

$$\forall z\,\forall w_1, \ldots, w_n\,\exists y\,\forall x\,(x \in y \leftrightarrow x \in z \wedge \phi).$$

AXIOM 4. *Pairing* (I §6).

$$\forall x\,\forall y\,\exists z\,(x \in z \wedge y \in z).$$

AXIOM 5. *Union* (I §6).

$$\forall \mathscr{F}\,\exists A\,\forall Y\,\forall x\,(x \in Y \wedge Y \in \mathscr{F} \rightarrow x \in A).$$

AXIOM 6. *Replacement Scheme* (I §6). For each formula ϕ with free variables among $x, y, A, w_1, \ldots, w_n$,

$$\forall A\,\forall w_1, \ldots, w_n\,[\forall x \in A\,\exists!y\,\phi \rightarrow \exists Y\,\forall x \in A\,\exists y \in Y\,\phi].$$

On the basis of Axioms 0, 1, 3, 4, 5 and 6, one may define $\subset$ (subset), 0 (empty set), S (ordinal successor; $S(x) = x \cup \{x\}$), and the notion of well-ordering. The following axioms are then defined.

AXIOM 7. *Infinity* (I §7).

$$\exists x\,(0 \in x \wedge \forall y \in x\,(S(y) \in x)).$$

AXIOM 8. *Power Set* (I §10).

$$\forall x\,\exists y\,\forall z\,(z \subset x \rightarrow z \in y).$$

Axiom 9. Choice

$$\forall A.\exists R.(R \text{ well-orders } A).$$